Journal of Statistical Physics, Vol. 74, Nos. 5/6, 1994

Ising Droplets in Five Dimensions

D. Stauffer[1]

Received October 20, 1993

Monte Carlo simulations indicate that the size distribution of Coniglio–Klein droplets in the five-dimensional nearest-neighbor Ising model corresponds to mean field Ising exponents.

KEY WORDS: Mean field theory; fluctuations, Coniglio–Klein clusters.

Droplet models for phase transitions are more than half a century old (nucleation theory, cluster expansions) and have worked successfully for the equilibrium properties of two- and three-dimensional Ising models.[1] The kinetics, however, is still problematic[2] in that we do not have a good picture of how decorrelation occur with clusters and this hampers our understanding of the cluster acceleration algorithms.[3] The best opportunity to understand the decorrelation mechanism is in mean field, but there is no verification that droplet models actually work at mean field critical points. Although they have been checked for spinodals in mean field in long-range interaction Ising models,[4] we are not aware of a confirmation of the relevance to the mean field Ising critical point, due to the difficulty with deconvoluting the critical from the spinodal data.

For dimensionalities d above the upper critical dimension of four, the leading critical exponents for Ising models take on their mean field values: $\beta = \nu = 1/2$, $\gamma = 1$, $\delta = 3$. What happens with the droplets in high dimensions: Are they described by these leading mean field exponents, or are they heterophase fluctuation *corrections* to the leading mean field behavior?

In both cases we expect them to follow the standard scaling law in zero field:

$$n(s) = s^{-\tau} f((T - T_c) s^{\sigma})$$

[1] Physics Department, St. F. X. University, Antigonish, Nova Scotia B2G 1CO, Canada. stauffer@px4.stfx.ca.

0022-4715/94/0300-1323$07.00/0

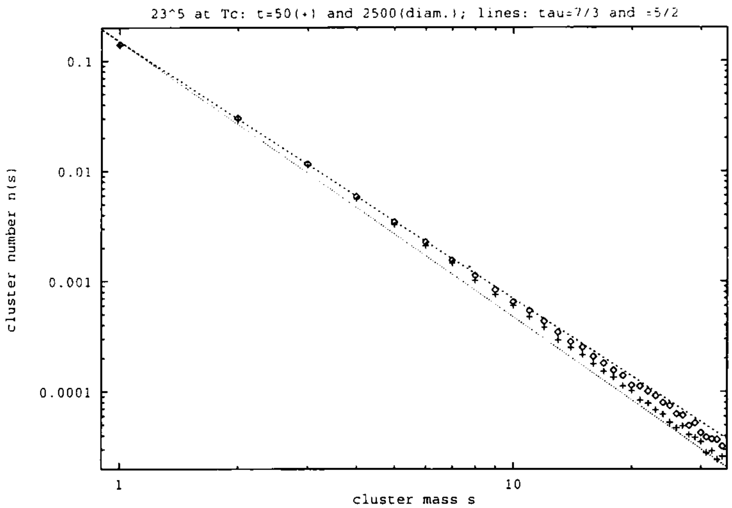

Fig. 1. Critical droplet numbers for 23^5 lattice versus cluster size s. The lines represent $\tau = 7/3$ and $5/2$.

with $\tau = 2 + 1/\delta = 7/3$ in the case of droplets as the leading term, and a largher Fisher exponent τ for the case of droplets as corrections. It is less clear how σ might change away from its three-dimensional value $1/\beta\delta$; thus, cluster counting right at the critical point and a determination of τ seems to be the best way to answer the above question of leading versus correction contribution of droplets. We use the Coniglio–Klein definition for clusters of overturned spins and work in $d = 5$, thereby avoiding the spinodal with $\tau = 5/2$ which comes in at $d = 6$.[5]

Figure 1 shows that for a rather small cluster (containing up to 34 sites) even right at the critical point the cluster numbers equilibrate within 10^3 time steps. We see that at T_c the power law decay is fulfilled, with $\tau = 2 + 1/\delta$ in both three and five dimensions; $\tau = 5/2$ works only for short times in $d = 5$. Below T_c, both spontaneous magnetization and the mass of the largest cluster vary as the square root of $T_c - T$; the results for the mean cluster size below T_c and the time dependence at T_c were less clear.

ACKNOWLEDGMENTS

We thank W. Klein for suggesting this test, the German–Israeli Foundation for cooperation, and the Canada Council for support.

REFERENCES

1. J. S. Wang, *Physica A* **161**:249 (1989); see also D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor and Francis, London, 1992), Chapter 7.
2. Z. Alexandrowicz, *J. Phys. A* **26**:L655 (1993).
3. J. S. Wang and R. H. Swendsen, *Physica A* **167**:565 (1990).
4. T. S. Ray and W. Klein, *J. Stat. Phys.* **61**:891 (1990).
5. T. S. Ray, *J. Stat. Phys.* **62**:463 (1991).